Recursive Formula in Arithmetic Sequences

Recursion is the process of choosing a starting term and repeatedly applying the same process to each term to arrive at the following term. Recursion requires that you know the value of the term immediately before the term you are trying to find.

A recursive formula always has two parts:

1. the **starting value** for a_1.
2. the **recursion equation** for a_n as a function of a_{n-1} (the term before it.)

Recursive formula:	Same recursive formula:
$a_1 = 4$ $a_n = 2a_{n-1}$	$a_1 = 4$ $a_{n+1} = 2a_n$

Examples:

1. Write the first four terms of the sequence: $a_1 = -4$, $a_n = a_{n-1} + 5$

$a_1 = -4$

$n = 2: \quad a_2 = a_{2-1} + 5 = 1$

$n = 3: \quad a_3 = a_{3-1} + 5 = 6$

$n = 4: \quad a_4 = a_{4-1} + 5 = 11$

In recursive formulas, each term is used to produce the next term. Follow the movement of the terms through the set up at the left.

Answer: -4, 1, 6, 11

2. Consider the sequence 2, 4, 6, 8, 10, ...

Explicit formula:
$a_n = 2n$

Recursive formula:
$a_1 = 2$
$a_n = a_{n-1} + 2$

Certain sequences, such as this arithmetic sequence, can be represented in more than one manner. This sequence can be represented as either an explicit (general) formula or a recursive formula.

3. Consider the sequence 3, 9, 27, 81, ...

Explicit formula:
$a_n = 3^n$

Recursive formula:
$a_1 = 3$
$a_n = 3a_{n-1}$

Certain sequences, such as this geometric sequence, can be represented in more than one manner. This sequence can be represented as either an explicit formula or a recursive formula.

4. Consider the sequence 2, 5, 26, 677, ...

Recursive formula:

$a_1 = 2$

$a_n = (a_{n-1})^2 + 1$

This sequence is neither arithmetic nor geometric. It does, however, have a pattern of development based upon each previous term.

5. Write the first 5 terms of the sequence $a_1 = 3$, $a_n = (-1)^n \cdot 5a_{n-1}$

$$a_1 = 3$$
$$a_2 = (-1)^2 \cdot 5a_{2-1} = 5 \cdot 3 = 15$$
$$a_3 = (-1)^3 \cdot 5a_{3-1} = (-1) \cdot 5 \cdot 15 = -75$$
$$a_4 = (-1)^4 \cdot 5a_{4-1} = 5 \cdot (-75) = -375$$
$$a_5 = (-1)^5 \cdot 5a_{5-1} = (-1) \cdot 5 \cdot (-375) = 1875$$

Notice how the value of *n* is used as the exponent for the value (-1). Also, remember that in recursive formulas, each term is used to produce the next term. Follow the movement of the terms through the set up at the left.

Answer: 3, 15, -75, -375, 1875

A sequence is an ordered list of numbers.

The **sum** of the terms of a sequence is called a **series.**

- Each number of a sequence is called a **term** (or element) of the sequence.
- A **finite sequence** contains a finite number of terms (you can count them). 1, 4, 7, 10, 13
- An **infinite sequence** contains an infinite number of terms (you cannot count them). 1, 4, 7, 10, 13, . . .
- The terms of a sequence are referred to in the subscripted form shown below, where the **natural number subscript** refers to the location (position) of the term in the sequence.

$$\underset{a_1}{1},\ \underset{a_2}{4},\ \underset{a_3}{7},\ \underset{a_4}{10},\ \underset{a_5}{13},\ \underset{a_6}{16},\ \ldots$$

(If you study computer programming languages such as C, C++, and Java, you will find that the first position in their arrays (sequences) start with a subscript of zero.)

- The **general form of a sequence** is represented: $a_1, a_2, a_3, \ldots a_n, \ldots$

- The **domain of a sequence** consists of the counting numbers 1, 2, 3, 4, ... and the **range** consists of the terms of the sequence.

- The terms in a sequence **may, or may not, have a pattern**, or a related formula. For some sequences, the terms are simply random.

Arithmetic Sequences

ADD

If a sequence of values follows a pattern of **adding a fixed amount** from one term to the next, it is referred to as an **arithmetic sequence**. The number added to each term is constant (always the same).

The fixed amount is called the **common difference, *d*,** referring to the fact that the difference between two successive terms yields the constant value that was added. To find the common difference, subtract the first term from the second term.