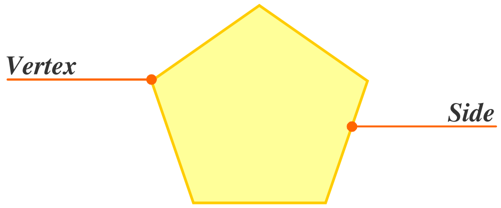
Interior and Exterior Angles in Polygons

Intro

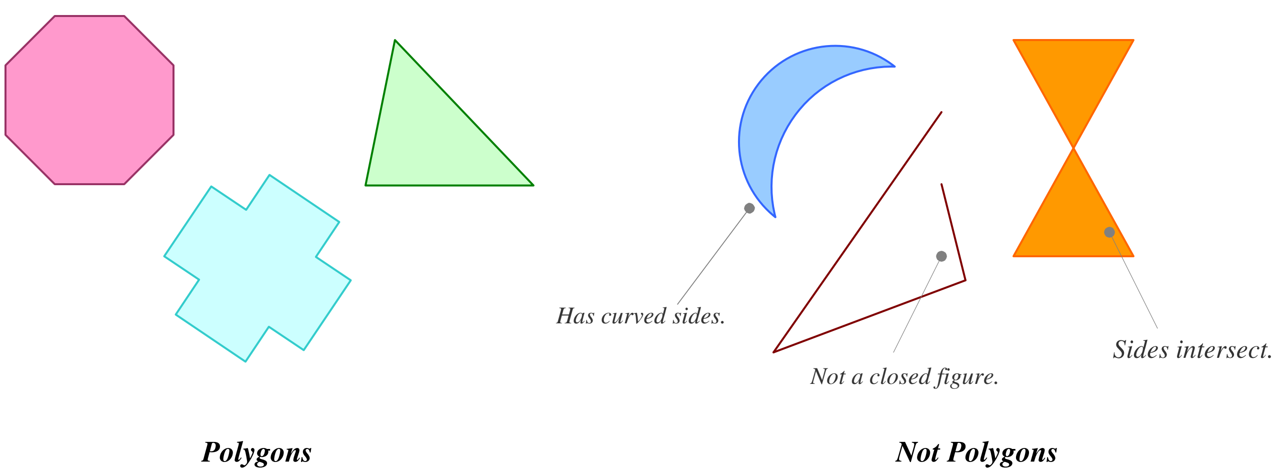
In order to understand how angles work in polygons, there are a few things you need to know about polygons themselves.

First, a **polygon** is a closed figure formed by joining line segments that meet only at their endpoints. The segments are called the **sides** of the polygon; each endpoint is called a **vertex**.



This is an example of a ***pentagon****—*
a polygon with five sides.

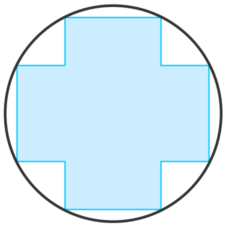
The following are example of polygons and non-polygons:



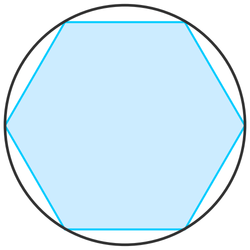
Concave and Convex

Polygons can either be **concave** or **convex**. The easiest way to explain the difference is by using a circle! If a circle is drawn around the polygon and **all** of its vertices are on the circle, the polygon is **convex**.

If just one of the vertices is **not on the circle**, the polygon is **concave**.



This is a **concave polygon** since just one of the vertices is not on the circle.



This is a **convex polygon** since all of the vertices are on the circle.

Classifying Polygons

Polygons are named by the number of sides they have. Since polygons come in many different shapes, you can classify them to simplify their description.

Polygons which have all sides and all angles congruent are called **regular polygons**.

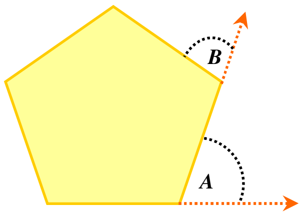
Here is a table classifying the most common polygons:

Name	Number of Sides	Name	Number of Sides
Triangle	3	Octagon	8
Quadrilateral	4	Nonagon	9
Pentagon	5	Decagon	10
Hexagon	6	Dodecagon	12
Heptagon	7	*n*-gon	*n*

Exterior Angles

An **exterior angle** of a polygon is the angle formed by a side and an extended, adjacent side.

The number of exterior angles in a polygon is equal to its number of sides. In the pentagon below, $\angle A$ and $\angle B$ are examples of exterior angles.

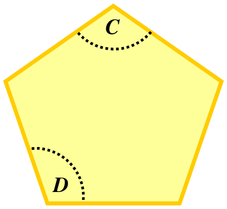


The sum of the measures of the exterior angles of a polygon is **360°**.

Interior angles

An **interior angle** of a polygon is the angle formed by two adjacent sides.

The number of interior angles in a polygon is equal to its number of sides. In the pentagon below, $\angle C$ and $\angle D$ are examples of interior angles.



The sum of the measures of the interior angles of a polygon is **180°(*n*-2)**, where *n* is the number of sides.

For example, to find the sum of the measures of a regular pentagon: 180(5 – 2)

180(3) = **540°**

To find more information on where this formula comes from, check this site out:

http://www.mathsisfun.com/geometry/interior-angles-polygons.html

Interior and Exterior Angles Problems

Example 1

Find the sum of the measures of the interior angles of an octagon.

Solution

Our shape is an octagon (8 sides) which means n = 8. Therefore, 180(8-2)

180 (6) = **1080°**

Example 2

Find the measure of one interior angle of a regular nonagon.

Solution

Our shape is a nonagon (9 sides) which means n = 9. Therefore, 180(9-2)

180(7) = **1260°**

Since we are looking for the measure of one interior angle, we need to divide this total by the number of sides (in this case, 9).

1260° ÷ 9 = 140°

*Note: You can **only** divide by the number of sides when you have a **regular** polygon!*

Example 3

If the sum of the measures of the interior angles of a polygon is 1800°, how many sides does the polygon have?

Solution

Since we are given the sum of the angle measures, we will use our formula in a slightly different manner: **180(n-2) = 1800**

180n – 360 = 1800 *Distribute 180 on the left-side.*

+360 +360 *Add 360 to each side to isolate n.*

180n = 2160

$$\frac{180n}{180} = \frac{2160}{180}$$ *Divide both sides by 180 to isolate n.*

n = 12, the shape has 12 sides (dodecagon).