# 14-2 Permutations and Combinations

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### Objectives

- Students will determine probabilities using permutations.
- Students will determine probabilities using combinations.

### Vocabulary

- <u>permutation</u> an arrangement or listing in which order is important. The number of permutations of n objects taken r at a time is:  ${}_{n}p_{r} = \frac{n!}{(n-r)!}$
- <u>combination</u> an arrangement or listing in which order is not important. The number of combinations of n objects taken r at a time is:  ${}_{n}C_{r} = \frac{n!}{(n-r)!r!}$

#### Example 1

Ms. Baraza asks pairs of students to go in front of her Spanish class to read statements in Spanish, and then to translate the statement into English. One student is the Spanish speaker and one is the English speaker. If Ms. Baraza has to choose between Jeff, Kathy, Guillermo, Ana, and Patrice, how many different ways can Ms. Baraza pair the students?

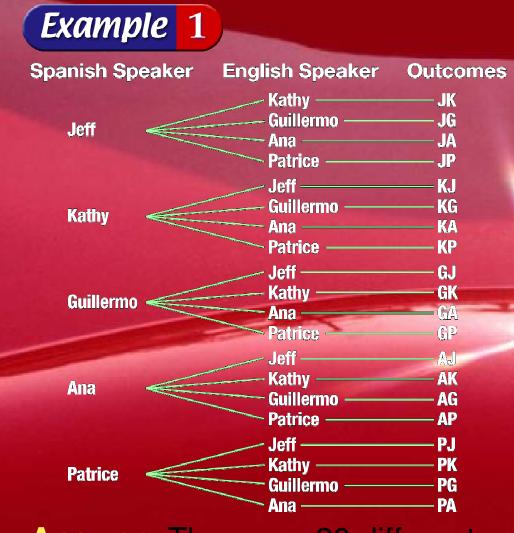
Use a tree diagram to show the possible arrangements.



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Answer: There are 20 different ways for the 5 students to be paired.





#### Your Turn

There are five finalists in the student art contest: Cal, Jeanette, Emily, Elizabeth, and Ron. The winner and the runner-up of the contest will receive prizes. How many possible ways are there for the winners to be chosen?

Answer: 20







Example 2

Find  $_{8}P_{4}$ .

 ${}_{8}P_{4}$ 

 $_{8}P_{4} = \frac{8!}{4!}$ 

$${}_{n}P_{r} = \frac{n!}{(n-r)!}$$

8!

(8-4)!

Definition of  $_{n}P_{r}$ 

#### Subtract.

m = \$, r =

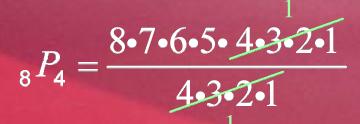


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Example 2



Definition of factorial

 $_{8}P_{4} = 8 \cdot 7 \cdot 6 \cdot 5 \text{ or } 1680$ 

Simplify.

### Answer: There are 1680 permutations of 8 objects taken 4 at a time.









Your Turn

Find  $_9P_5$ .

#### **Answer:** 15,120







#### *Example <mark>3a</mark>*

Shaquille has a 5-digit pass code to access his e-mail account. The code is made up of the even digits 2, 4, 6, 8, and 0. Each digit can be used only once.

How many different pass codes could Shaquille have?

Since the order of the numbers in the code is important, this situation is a permutation of 5 digits taken 5 at a time.

$${}_{n}P_{r}=\frac{n!}{(n-r)!}$$

Definition of permutation

$$_{5}P_{5} = \frac{5!}{(5-5)!}$$

n = 5, r = 5



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Example <mark>3a</mark>

# ${}_{5}P_{5} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1}$ or 120

#### Definition of factorial

## Answer: There are 120 possible pass codes with the digits 2, 4, 6, 8, and 0.







#### Example 3b

Shaquille has a 5-digit pass code to access his e-mail account. The code is made up of the even digits 2, 4, 6, 8, and 0. Each digit can be used only once.

What is the probability that the first two digits of his code are both greater than 5?

Use the Fundamental Counting Principle to determine the number of ways for the first two digits to be greater than 5.

- There are 2 digits greater than 5 and 3 digits less than 5.
- The number of choices for the first two digits, if they are greater than 5, is 2 • 1.
- The number of choices for the remaining digits is 3 • 2 • 1.



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#### Example 3b

 The number of favorable outcomes is 2 • 1 • 3 • 2 • 1 or 12. There are 12 ways for this event to occur out of the 120 possible permutations.

 $P(\text{first } 2 \text{ digits} > 5) = \frac{12}{120} \leftarrow \text{number of favorable outcomes}$ 

 $=\frac{10}{10}$  Simplify.

Answer: The probability that the first two digits of the

pass code are greater than 5 is  $\frac{1}{10}$  or 10%.





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#### Your Turn

Bridget and Brittany are trying to find a house, but they cannot remember the address. They can remember only that the digits used are 1, 2, 5, and 8, and that no digit is used twice.

a. How many possible addresses are there?
Answer: 24 addresses

b. What is the probability that the first two numbers are odd?

Answer:  $\frac{1}{6}$  or about 17%





#### Example 4

#### **Multiple-Choice Test Item**

Customers at Tony's Pizzeria can choose 4 out of 12 toppings for each pizza for no extra charge. How many different combinations of pizza toppings can be chosen?

A 495B 792C 11,880D 95,040

#### **Read the Test Item**

The order in which the toppings are chosen does not matter, so this situation represents a combination of 12 toppings taken 4 at a time.



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#### Example 4

#### **Solve the Test Item**

$$_{n}C_{r} = \frac{n!}{(n-r)!r!}$$

$$_{12}C_4 = \frac{12!}{(12-4)!4!}$$

12•11•10•9•8•7•6•5•4•3•2•1 8•7•6•5•4•3•2•1•4•3•2•1 Definition of combination

$$n = 12, r = 4$$

Definition of factorial



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Lesson 14-2







 $=\frac{12\cdot11\cdot10\cdot9}{4\cdot3\cdot2\cdot1} \text{ or } 495$ 

Simplify.

#### Answer: There are 495 different ways to select toppings. Choice A is correct.







#### Your Turn

#### **Multiple-Choice Test Item**

A cable company is having a sale on their premium channels. Out of 8 possible premium channels, they are allowing customers to pick 5 channels at no extra charge. How many channel packages are there?

**A** 6720 **C** 336

Answer: B





**B** 56

**D** 120

#### Example <mark>5</mark>a

Diane has a bag full of coins. There are 10 pennies, 6 nickels, 4 dimes, and 2 quarters in the bag.

How many different ways can Diane pull four coins out of the bag?

The order in which the coins are chosen does not matter, so we must find the number of combinations of 22 coins taken 4 at a time.

**5-Minute Check** 

$${}_{n}C_{r} = \frac{n!}{(n-r)!r!}$$

Definition of combination

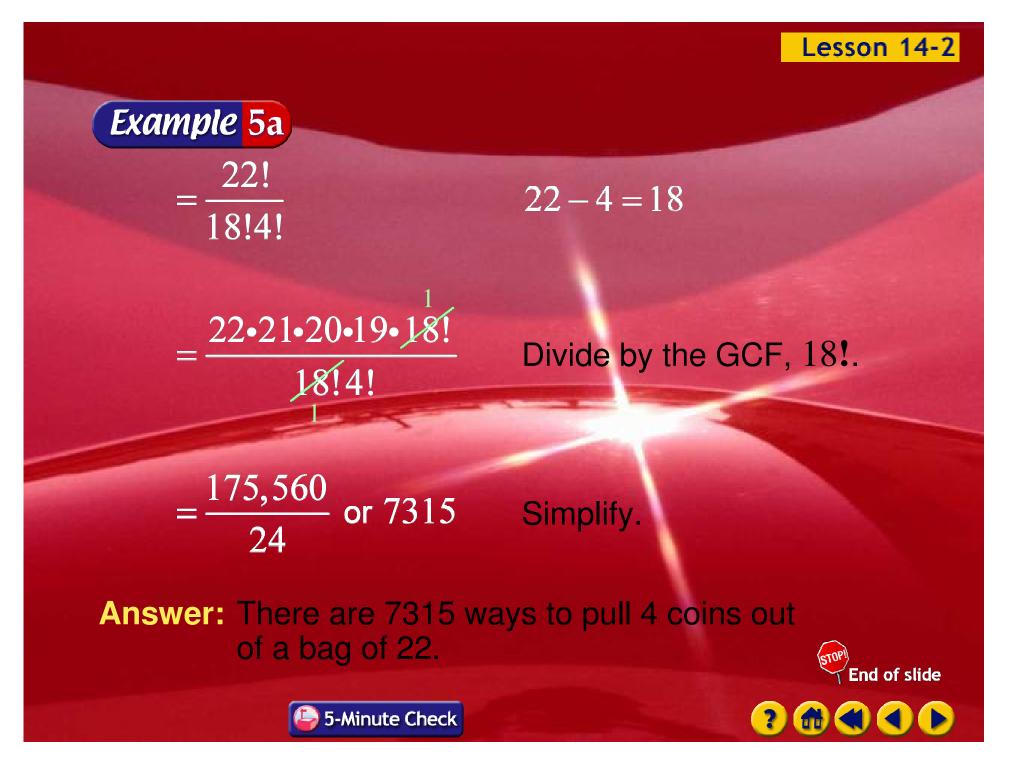
$$_{22}C_4 = \frac{22!}{(22-4)!4!}$$

n = 22, r = 4



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#### Example 5b

Diane has a bag full of coins. There are 10 pennies, 6 nickels, 4 dimes, and 2 quarters in the bag.

What is the probability that she will pull two pennies and two nickels out of the bag?

There are two questions to consider.

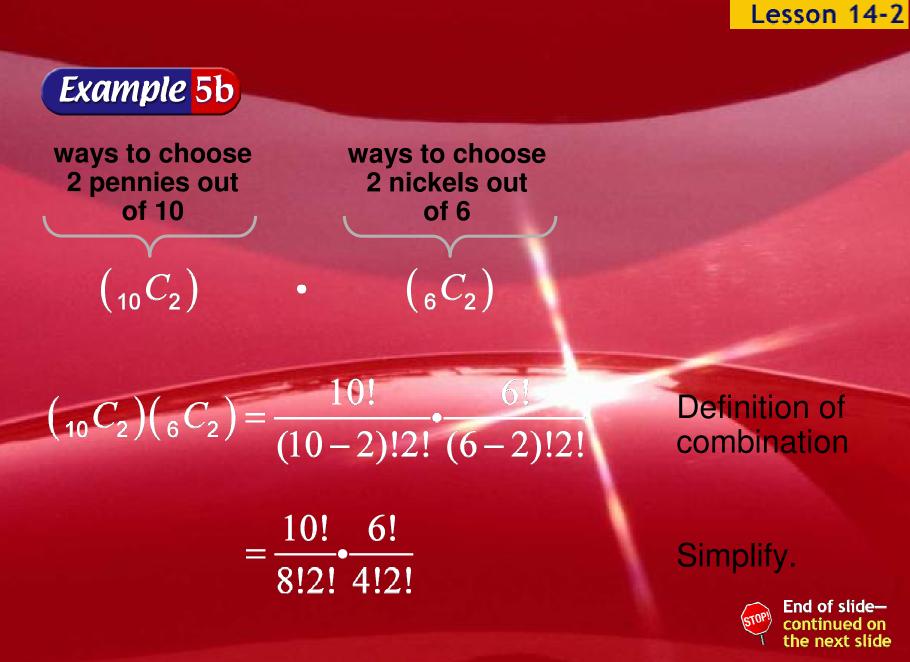
- How many ways can 2 pennies be pulled from 10?
  - How many ways can 2 nickels be pulled from 6?

Using the Fundamental Counting Principle, the answer can be determined with the product of the two combinations.



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Divide the first term by its GCF, 8!, and the second term by its GCF, 4!.



10•9 6•5

2! 2!

Simplify.

### There are 675 ways to choose this particular combination out of 7315 possible combinations.



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#### Example 5b

# $P(2 \text{ pennies}, 2 \text{ nickels}) = \frac{675}{7315} \leftarrow \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$

# $=\frac{135}{1463}$ Simplify.

# Answer: The probability that Diane will select two pennies and two nickels is $\frac{135}{1463}$ , or about 9%.







#### Your Turn

At a factory, there are 10 union workers, 12 engineers, and 5 foremen. The company needs 6 of these workers to attend a national conference.

a. How many ways could the company choose the 6 workers?

Answer: 296,010 ways

 If the workers are chosen randomly, what is the probability that 3 union workers, 2 engineers, and 1 foreman are selected?

Answer:  $\frac{40}{299}$  or about 13%



