14-2 Permutations and Combinations

Objectives

- Students will determine probabilities using permutations.
- Students will determine probabilities using combinations.

Vocabulary

- permutation – an arrangement or listing in which order is important. The number of permutations of n objects taken r at a time is:

$$ {}_{n}p_{r} = \frac{n!}{(n-r)!} $$

- combination – an arrangement or listing in which order is not important. The number of combinations of n objects taken r at a time is:

$$ {}_{n}C_{r} = \frac{n!}{(n-r)!r!} $$

Example 1

Ms. Baraza asks pairs of students to go in front of her Spanish class to read statements in Spanish, and then to translate the statement into English. One student is the Spanish speaker and one is the English speaker. If Ms. Baraza has to choose between Jeff, Kathy, Guillermo, Ana, and Patrice, how many different ways can Ms. Baraza pair the students?

Use a tree diagram to show the possible arrangements.

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Example 1

Spanish Speaker	English Speaker	Outcomes
Jeff	Kathy	JK
	Guillermo	JG
	Ana	JA
	Patrice	JP
Kathy	Jeff	KJ
	Guillermo	KG
	Ana	KA
	Patrice	KP
Guillermo	Jeff	GJ
	Kathy	GK
	Ana	GA
	Patrice	GP
Ana	Jeff	AJ
	Kathy	AK
	Guillermo	AG
	Patrice	AP
Patrice	Jeff	PJ
	Kathy	PK
	Guillermo	PG
	Ana	PA

Answer: There are 20 different ways for the 5 students to be paired.

Your Turn

There are five finalists in the student art contest: Cal, Jeanette, Emily, Elizabeth, and Ron. The winner and the runner-up of the contest will receive prizes. How many possible ways are there for the winners to be chosen?

Answer: 20

Example 2

Find $_{8}P_{4}$.

$$_{n}P_{r} = \frac{n!}{(n-r)!}$$ Definition of $_{n}P_{r}$

$$_{8}P_{4} = \frac{8!}{(8-4)!}$$ $n = 8, r = 4$

$$_{8}P_{4} = \frac{8!}{4!}$$ Subtract.

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Example 2

$$_8P_4 = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}^{1}}{\cancel{4 \cdot 3 \cdot 2 \cdot 1}_{1}}$$ Definition of factorial

$$_8P_4 = 8 \cdot 7 \cdot 6 \cdot 5 \text{ or } 1680$$ Simplify.

Answer: There are 1680 permutations of 8 objects taken 4 at a time.

Your Turn

Find ${}_{9}P_{5}$.

Answer: 15,120

Example 3a

Shaquille has a 5-digit pass code to access his e-mail account. The code is made up of the even digits 2, 4, 6, 8, and 0. Each digit can be used only once.

How many different pass codes could Shaquille have?

Since the order of the numbers in the code is important, this situation is a permutation of 5 digits taken 5 at a time.

$${}_nP_r = \frac{n!}{(n-r)!}$$ Definition of permutation

$${}_5P_5 = \frac{5!}{(5-5)!}$$ $n = 5, r = 5$

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Example 3a

$$_5P_5 = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1} \text{ or } 120$$ Definition of factorial

Answer: There are 120 possible pass codes with the digits 2, 4, 6, 8, and 0.

Example 3b

Shaquille has a 5-digit pass code to access his e-mail account. The code is made up of the even digits 2, 4, 6, 8, and 0. Each digit can be used only once.

What is the probability that the first two digits of his code are both greater than 5?

Use the Fundamental Counting Principle to determine the number of ways for the first two digits to be greater than 5.

- There are 2 digits greater than 5 and 3 digits less than 5.
- The number of choices for the first two digits, if they are greater than 5, is 2 • 1.
- The number of choices for the remaining digits is 3 • 2 • 1.

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Example 3b

- The number of favorable outcomes is 2 • 1 • 3 • 2 • 1 or 12. There are 12 ways for this event to occur out of the 120 possible permutations.

$$P(\text{first } 2 \text{ digits} > 5) = \frac{12}{120} \begin{matrix} \leftarrow \text{number of favorable outcomes} \\ \leftarrow \text{number of possible outcomes} \end{matrix}$$

$$= \frac{1}{10}$$ Simplify.

Answer: The probability that the first two digits of the pass code are greater than 5 is $\frac{1}{10}$ or 10%.

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Your Turn

Bridget and Brittany are trying to find a house, but they cannot remember the address. They can remember only that the digits used are 1, 2, 5, and 8, and that no digit is used twice.

a. How many possible addresses are there?

Answer: 24 addresses

b. What is the probability that the first two numbers are odd?

Answer: $\frac{1}{6}$ or about 17%

Example 4

Multiple-Choice Test Item

Customers at Tony's Pizzeria can choose 4 out of 12 toppings for each pizza for no extra charge. How many different combinations of pizza toppings can be chosen?

A 495 **B** 792
C 11,880 **D** 95,040

Read the Test Item

The order in which the toppings are chosen does not matter, so this situation represents a combination of 12 toppings taken 4 at a time.

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Example 4

Solve the Test Item

$${}_{n}C_{r} = \frac{n!}{(n-r)!r!}$$ Definition of combination

$${}_{12}C_{4} = \frac{12!}{(12-4)!4!}$$ $n = 12, r = 4$

$$= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot \overset{1}{\cancel{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}}{\underset{1}{\cancel{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$ Definition of factorial

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Example 4

$$= \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1} \text{ or } 495$$ Simplify.

Answer: There are 495 different ways to select toppings. Choice A is correct.

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Your Turn

Multiple-Choice Test Item

A cable company is having a sale on their premium channels. Out of 8 possible premium channels, they are allowing customers to pick 5 channels at no extra charge. How many channel packages are there?

A 6720 **B** 56

C 336 **D** 120

Answer: B

Example 5a

Diane has a bag full of coins. There are 10 pennies, 6 nickels, 4 dimes, and 2 quarters in the bag.

How many different ways can Diane pull four coins out of the bag?

The order in which the coins are chosen does not matter, so we must find the number of combinations of 22 coins taken 4 at a time.

$${}_{n}C_{r} = \frac{n!}{(n-r)!r!}$$ Definition of combination

$${}_{22}C_{4} = \frac{22!}{(22-4)!4!}$$ $n = 22, r = 4$

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Example 5a

$$= \frac{22!}{18!4!}$$ $22 - 4 = 18$

$$= \frac{22 \cdot 21 \cdot 20 \cdot 19 \cdot \overset{1}{\cancel{18!}}}{\underset{1}{\cancel{18!}}4!}$$ Divide by the GCF, $18!$.

$$= \frac{175{,}560}{24} \text{ or } 7315$$ Simplify.

Answer: There are 7315 ways to pull 4 coins out of a bag of 22.

Example 5b

Diane has a bag full of coins. There are 10 pennies, 6 nickels, 4 dimes, and 2 quarters in the bag.

What is the probability that she will pull two pennies and two nickels out of the bag?

There are two questions to consider.

- How many ways can 2 pennies be pulled from 10?
- How many ways can 2 nickels be pulled from 6?

Using the Fundamental Counting Principle, the answer can be determined with the product of the two combinations.

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Example 5b

ways to choose 2 pennies out of 10 · ways to choose 2 nickels out of 6

$$\left({}_{10}C_2\right) \cdot \left({}_{6}C_2\right)$$

$$\left({}_{10}C_2\right)\left({}_{6}C_2\right) = \frac{10!}{(10-2)!2!} \cdot \frac{6!}{(6-2)!2!}$$ Definition of combination

$$= \frac{10!}{8!2!} \cdot \frac{6!}{4!2!}$$ Simplify.

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Example 5b

$$= \frac{10 \cdot 9}{2!} \cdot \frac{6 \cdot 5}{2!}$$

Divide the first term by its GCF, 8!, and the second term by its GCF, 4!.

$$= 675$$

Simplify.

There are 675 ways to choose this particular combination out of 7315 possible combinations.

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5-Minute Check

Example 5b

$$P(2 \text{ pennies, } 2 \text{ nickels}) = \frac{675}{7315} \quad \frac{\leftarrow \text{number of favorable outcomes}}{\leftarrow \text{number of possible outcomes}}$$

$$= \frac{135}{1463} \quad \text{Simplify.}$$

Answer: The probability that Diane will select two pennies and two nickels is $\frac{135}{1463}$, or about 9%.

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Your Turn

At a factory, there are 10 union workers, 12 engineers, and 5 foremen. The company needs 6 of these workers to attend a national conference.

a. How many ways could the company choose the 6 workers?

Answer: 296,010 ways

b. If the workers are chosen randomly, what is the probability that 3 union workers, 2 engineers, and 1 foreman are selected?

Answer: $\frac{40}{299}$ or about 13%

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